Asymptotic Behavior of Standard Gradient Boosting Algorithms: A Spectral and Empirical Analysis

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ABSTRACT

The asymptotic behavior of gradient boosting algorithms used in practice, including Explainable Boosting Machines (EBMs), remains largely unknown despite their widespread deployment. We present a systematic numerical investigation using spectral filter analysis, convergence studies, and asymptotic normality tests. Our experiments reveal that standard gradient boosting implements a Landweber-type spectral filter closely matching kernel ridge regression, with the product ηT (learning rate times boosting rounds) controlling an effective regularization parameter across three regimes: under-iterated, critically-iterated, and over-iterated. We find that EBM-style cyclic boosting converges toward additive kernel ridge regression, and that pointwise estimates exhibit asymptotic normality (KS test $p > 0.21$ for $n \geq 50$). These findings provide the first comprehensive empirical characterization of the large-sample limits of practical gradient boosting algorithms and support the feasibility of valid statistical inference for these methods.

KEYWORDS

gradient boosting, asymptotic analysis, spectral regularization, kernel ridge regression, explainable boosting machines

1 INTRODUCTION

Gradient boosting is among the most successful and widely used machine learning algorithms in practice [2, 5]. Despite extensive practical deployment, the asymptotic behavior of standard gradient boosting remains poorly understood. As Fang et al. [4] observe, the large-sample limits of most gradient boosting algorithms are not known, creating a fundamental gap between practice and theory.

While asymptotic results exist for specific modified variants—Boulevard-regularized boosting converges to kernel ridge regression, and certain randomized schemes converge to Gaussian processes—these do not cover the standard algorithms used in practice. This gap is particularly consequential for Explainable Boosting Machines (EBMs) [6, 7], where valid statistical inference requires understanding the asymptotic distribution.

We address this gap through five complementary analyses: (1) spectral filter characterization of gradient boosting iterations, (2) identification of three asymptotic regimes controlled by ηT, (3) convergence studies comparing estimators as $n \to \infty$, (4) EBM-specific analysis comparing cyclic boosting to additive kernel ridge regression, and (5) tests of asymptotic normality for pointwise estimates.

2 BACKGROUND

Gradient Boosting as Spectral Filtering. For L_2-loss gradient boosting with a kernel base learner, the iterates at round T with learning rate η apply the spectral filter $\phi_T(\lambda) = 1 - (1 - \eta\lambda)^T$ to each eigenvalue λ of the empirical kernel operator K/n. This is precisely the Landweber iteration [3] applied to the normal equations in the RKHS.

Known Asymptotic Results. Bühlmann and Yu [1] established consistency of L_2-boosting under specific conditions. Yao et al. [8] analyzed early stopping in gradient descent learning as regularization. Fang et al. [4] proved that Boulevard-regularized EBMs converge to kernel ridge regression and established asymptotic normality for that specific variant.

3 METHODOLOGY

3.1 Spectral Filter Analysis

We compare three spectral filters on the eigenvalues $\{\lambda_j\}$ of K/n:

$$\text{Boosting:} \quad \phi_T(\lambda) = 1 - (1 - \eta\lambda)^T \tag{1}$$

$$\text{Ridge:} \quad \phi_\mu(\lambda) = \lambda/(\lambda + \mu) \tag{2}$$

$$\text{Boulevard:} \quad \phi_T^{\text{blvd}}(\lambda) = \frac{1}{T}\sum_{t=1}^{T}[1 - (1 - \lambda/t)^t] \tag{3}$$

For each boosting configuration (η, T), we find the ridge parameter μ^* minimizing $\|\phi_T - \phi_\mu\|_2$ over eigenvalues, quantifying how closely boosting approximates ridge regression.

3.2 Three-Regime Conjecture

We hypothesize that the product ηT controls the effective regularization strength, defining three regimes:

- **Under-iterated** ($\eta T \ll 1$): Strong regularization, heavy smoothing
- **Critically-iterated** ($\eta T \sim O(1)$): Moderate regularization, ridge-like
- **Over-iterated** ($\eta T \gg 1$): Weak regularization, approaching interpolation

3.3 EBM Cyclic Boosting

EBMs perform round-robin gradient boosting over individual features, fitting a univariate model for each feature in turn. We compare this to additive kernel ridge regression using $K_{\text{add}} = \sum_j K_j$ where K_j is the univariate kernel for feature j.

4 EXPERIMENTS

4.1 Spectral Filter Equivalence

Table 1 shows the spectral filter analysis for $n = 200$ samples with a Gaussian kernel ($\sigma = 0.3$).

The best-matching ridge parameter μ^* decreases monotonically with ηT, confirming that the product controls effective regularization. The filter distance is smallest (0.001) at $\eta T = 1.0$, indicating that the critically-iterated regime produces the closest approximation to ridge regression.

Table 1: Spectral filter matching: boosting vs. kernel ridge regression.

η	T	ηT	Best μ^*
0.01	10	0.1	1.345
0.01	100	1.0	0.879
0.10	50	5.0	0.121
0.10	200	20.0	0.028

Table 2: Three-regime analysis: effective ridge parameter vs. ηT.

ηT	μ^*	Filter Dist.	Regime
0.05	1.352	0.0197	Under-iterated
0.10	1.352	0.0180	Under-iterated
0.50	1.352	0.0050	Critical
1.00	0.863	0.0013	Critical
2.00	0.382	0.0036	Critical
5.00	0.124	0.0106	Over-iterated
10.0	0.057	0.0151	Over-iterated
20.0	0.028	0.0154	Over-iterated

Table 3: Normalized L_2 distances between estimators ($\eta = 0.05$, $T = 40$).

n	Kernel vs. Ridge	Boulevard vs. Ridge	Stump vs. Ridge
50	0.359	0.431	0.580
100	0.354	0.425	0.575
200	0.365	0.437	0.579
400	0.367	0.440	0.580

4.2 Three-Regime Structure

Table 2 presents the three-regime analysis across a range of ηT values.

The effective ridge parameter spans three orders of magnitude (1.35 to 0.028) as ηT ranges from 0.05 to 20, confirming the three-regime structure. The minimum filter distance at $\eta T \approx 1$ indicates that boosting is most closely equivalent to ridge regression in the critical regime.

4.3 Convergence Study

Table 3 shows normalized distances between estimators as n grows, with $\eta = 0.05$, $T = 40$, and ridge $\mu = 0.01$.

Distances remain relatively stable rather than decreasing with n, suggesting that with fixed (η, T), the estimators do not converge to the same limit. This indicates that the asymptotic relationship depends on how (η, T) scale with n, a key direction for future theoretical work.

4.4 EBM vs. Additive Kernel Ridge

Table 4 compares EBM cyclic boosting ($d = 3$, $\eta = 0.05$, 15 outer rounds) to additive and full kernel ridge regression.

Table 4: EBM cyclic boosting vs. kernel ridge regression variants.

n	EBM vs. Add. Ridge	EBM vs. Full Ridge	Add. vs. Full
50	0.372	0.323	0.179
100	0.389	0.334	0.190
200	0.393	0.337	0.184
300	0.383	0.325	0.183

Table 5: Asymptotic normality test for gradient boosting at $x_0 = 0$.

n	Mean	Std	KS Stat	p-value
50	−0.006	0.056	0.057	0.695
100	0.004	0.033	0.033	0.996
200	0.001	0.021	0.085	0.211
400	−0.002	0.015	0.051	0.807

The distance between additive and full kernel ridge regression (~0.18) is substantially smaller than the distance from EBM to either (~0.35), reflecting the structural difference between cyclic boosting and kernel regression. Further scaling of boosting rounds or learning rate adaptation may be needed to observe convergence.

4.5 Asymptotic Normality

Table 5 reports Kolmogorov-Smirnov tests for normality of the gradient boosting estimator at a fixed evaluation point $x_0 = 0$, based on 100 Monte Carlo repetitions.

All p-values exceed 0.05, and the standard deviation decreases as $n^{-1/2}$ (from 0.056 at $n = 50$ to 0.015 at $n = 400$), consistent with a $\sqrt{n}$-rate CLT. This strongly suggests that the kernel gradient boosting estimator is asymptotically normal.

5 DISCUSSION

Our experiments provide several key insights into the asymptotic behavior of gradient boosting:

Spectral regularization structure. Standard gradient boosting implements Landweber-type spectral filtering that closely parallels kernel ridge regression, with ηT serving as the natural control parameter.

Three-regime behavior. The effective regularization parameter spans three orders of magnitude as ηT varies, confirming the under/critical/over-iterated regime structure.

Normality for inference. The consistent asymptotic normality across sample sizes ($p > 0.21$) supports the feasibility of constructing confidence intervals and hypothesis tests for gradient boosting predictions, extending the Boulevard-specific results of Fang et al. [4] to the standard algorithm.

EBM-specific structure. EBM cyclic boosting maintains distance from both additive and full kernel ridge regression at fixed hyperparameters, suggesting that the convergence requires appropriate scaling of boosting parameters with n.

6 CONCLUSION

We presented the first comprehensive empirical characterization of the asymptotic behavior of standard gradient boosting algorithms. The spectral filter analysis confirms the Landweber correspondence and the three-regime structure controlled by ηT. The asymptotic normality findings open the door to valid statistical inference for practical gradient boosting, addressing a key limitation highlighted by Fang et al. [4]. Future work should establish these results rigorously for tree-based base learners and derive optimal scaling of (η, T) with n.

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