Numerical Investigation of Non-Ergodic States in Higher-Dimensional Disordered Quantum Systems

Anonymous Author(s)

ABSTRACT

We address the open question of whether non-ergodic states exist in many-body systems beyond one spatial dimension by performing exact diagonalization of the disordered Heisenberg model on 1D chains ($L = 8, 10$) and 2D square lattices (2×2, 3×3). We compute the adjacent gap ratio $\langle r \rangle$, fractal dimension D_2, and entanglement entropy across disorder strengths $W \in [0.5, 15]$. In 1D, the gap ratio decreases from 0.400 at $W = 0.5$ to 0.379 at $W = 15$ for $L = 10$, approaching the Poisson value ($r_{\text{Poi}} = 0.386$) but never reaching the GOE value ($r_{\text{GOE}} = 0.531$). The fractal dimension decreases monotonically from $D_2 = 0.58$ to $D_2 = 0.08$, indicating progressive localization. In 2D (3×3, $N = 9$ sites), gap ratios remain near 0.39–0.41 across all disorder strengths, with D_2 decreasing from 0.50 to 0.18. No evidence for a distinct non-ergodic extended (NEE) phase is found at the accessible system sizes. The average gap ratio at moderate disorder ($W = 3$–5) is $\langle r \rangle = 0.391$ (1D) and 0.394 (2D), both intermediate between Poisson and GOE limits, consistent with finite-size crossover effects rather than a stable NEE phase.

1 INTRODUCTION

Many-body localization (MBL) in one dimension is theoretically established [1, 2, 7] and experimentally observed. Whether MBL or non-ergodic extended (NEE) states survive in dimensions $D > 1$ is debated [3, 5]. NEE phases, characterized by multifractal eigenstates that are neither fully extended nor localized, appear in random matrix models [4] but their existence in realistic finite-dimensional systems remains unresolved.

We perform exact diagonalization of the disordered XXZ Heisenberg model in 1D and 2D to search for signatures of NEE behavior through level statistics, participation ratios, and entanglement entropy.

1.1 Related Work

The MBL phase transition was first characterized through level statistics [6]. Local integrals of motion provide the theoretical framework for MBL [8]. De Roeck and Huveneers [3] argued MBL is unstable in $D > 1$, while Lunkin et al. [5] recently reported evidence for a 2D quantum glass state.

2 METHODS

2.1 Model Hamiltonian

We study the XXZ Heisenberg model with random on-site disorder:

$$H = J_{xx} \sum_{\langle i,j \rangle} (S_i^x S_j^x + S_i^y S_j^y) + J_{zz} \sum_{\langle i,j \rangle} S_i^z S_j^z + \sum_i h_i S_i^z \tag{1}$$

where $h_i \in [-W/2, W/2]$ are uniform random fields, $J_{xx} = J_{zz} = 1$, and the sums run over nearest-neighbor pairs with periodic boundary conditions. In 1D, $N = L$ sites; in 2D, $N = L \times L$ sites on a square lattice.

Table 1: Gap ratio $\langle r \rangle$ vs disorder strength (largest systems).

W	1D ($L = 10$)	2D (3×3)
0.5	0.400	0.407
1.0	0.396	0.408
3.0	0.392	0.388
5.0	0.390	0.399
8.0	0.393	0.393
10.0	0.384	0.406
15.0	0.379	0.405

Table 2: Fractal dimension D_2 vs disorder strength.

W	1D ($L = 10$)	2D (3×3)
0.5	0.576	0.500
3.0	0.433	0.488
5.0	0.278	0.442
10.0	0.128	0.293
15.0	0.083	0.178

2.2 Diagnostics

The adjacent gap ratio $r_n = \min(\delta_n, \delta_{n+1})/\max(\delta_n, \delta_{n+1})$ distinguishes GOE statistics ($\langle r \rangle = 0.531$, ergodic) from Poisson statistics ($\langle r \rangle = 0.386$, localized) [6]. The fractal dimension $D_2 = -\log(\text{IPR})/\log(N)$ distinguishes extended ($D_2 \to 1$), multifractal ($0 < D_2 < 1$), and localized ($D_2 \to 0$) states. Entanglement entropy S provides additional characterization: volume law (ergodic), area law (localized), or intermediate scaling (NEE).

3 RESULTS

3.1 Gap Ratio Analysis

Table 1 presents gap ratios at the largest system sizes. In 1D, $\langle r \rangle$ decreases monotonically from 0.400 to 0.379, approaching but not reaching the Poisson limit. In 2D, values remain near 0.39–0.41 with no clear trend, suggesting the system is too small to resolve the transition.

3.2 Fractal Dimension

Table 2 shows D_2 decreases with disorder in both dimensions, consistent with progressive localization. The 2D system shows higher D_2 values at the same disorder strength, consistent with enhanced delocalization from additional connectivity.

3.3 Phase Boundary Analysis

Using the midpoint criterion $r_{\text{mid}} = 0.459$ between GOE and Poisson, no system size exhibits $\langle r \rangle > r_{\text{mid}}$, so no ergodic-to-NEE transition is detected. The NEE width is 0.0 for both 1D and 2D at all system sizes studied. The average gap ratio at moderate disorder ($W = 3$–5) is 0.391 (1D) and 0.394 (2D).

4 CONCLUSION

Our exact diagonalization study finds no evidence for a distinct NEE phase at the accessible system sizes ($L \leq 10$ in 1D, 3×3 in 2D). Gap ratios remain near the Poisson value across all disorder strengths, and fractal dimensions decrease monotonically, consistent with a crossover from weakly localized to strongly localized behavior without an intermediate extended phase. The 2D system shows slightly higher D_2 values, hinting at enhanced delocalization, but system sizes are too small to draw conclusions about the thermodynamic limit. Larger-scale studies using tensor network methods or quantum simulation on programmable processors are needed to resolve this open question.

4.1 Limitations and Ethical Considerations

The primary limitation is the small system sizes accessible to exact diagonalization (2^N scaling of Hilbert space dimension). For 2D, the largest system ($3 \times 3 = 9$ sites) is far from the thermodynamic limit. Finite-size effects strongly affect level statistics at these sizes. No ethical concerns arise from this computational physics study.

REFERENCES

[1] Dmitry A. Abanin, Ehud Altman, Immanuel Bloch, and Maksym Serbyn. 2019. Colloquium: Many-body localization, thermalization, and entanglement. *Reviews of Modern Physics* 91 (2019), 021001.
[2] D. M. Basko, I. L. Aleiner, and B. L. Altshuler. 2006. Metal–insulator transition in a weakly interacting many-electron system with localized single-particle states. *Annals of Physics* 321 (2006), 1126–1205.
[3] Wojciech De Roeck and François Huveneers. 2017. Stability and instability towards delocalization in many-body localization systems. *Physical Review B* 95 (2017), 155129.
[4] V. E. Kravtsov, I. M. Khaymovich, E. Cuevas, and M. Amini. 2015. A random matrix model with localization and ergodic transitions. *New Journal of Physics* 17 (2015), 122002.
[5] A. Lunkin et al. 2026. Evidence for a two-dimensional quantum glass state at high temperatures. *arXiv preprint arXiv:2601.01309* (2026).
[6] Vadim Oganesyan and David A. Huse. 2007. Localization of interacting fermions at high temperature. *Physical Review B* 75 (2007), 155111.
[7] Arijeet Pal and David A. Huse. 2010. Many-body localization phase transition. *Physical Review B* 82 (2010), 174411.
[8] Maksym Serbyn, Zlatko Papić, and Dmitry A. Abanin. 2013. Local conservation laws and the structure of the many-body localized states. *Physical Review Letters* 111 (2013), 127201.