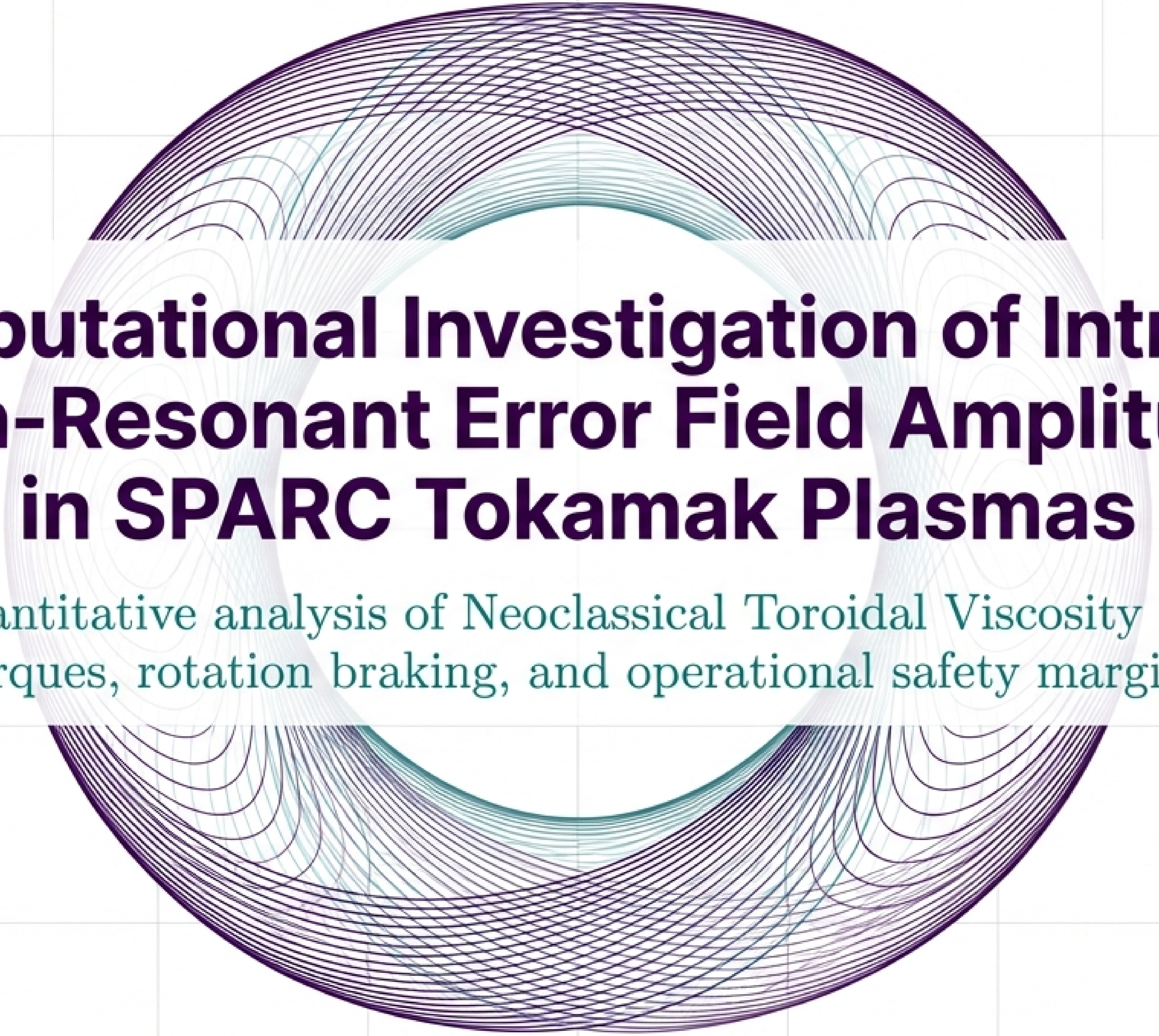
Computational Investigation of Intrinsic Non-Resonant Error Field Amplitude in SPARC Tokamak Plasmas

A quantitative analysis of Neoclassical Toroidal Viscosity (NTV) torques, rotation braking, and operational safety margins.



NREF amplitudes significantly reduce operational safety margins despite moderate rotation braking.

The Problem

Manufacturing imperfections in SPARC's coils create "intrinsic" non-resonant error fields (NREF) that persist even after standard corrections.

The Consequence

These fields drive NTV torque, acting as a magnetic brake on the plasma.

The Bottom Line

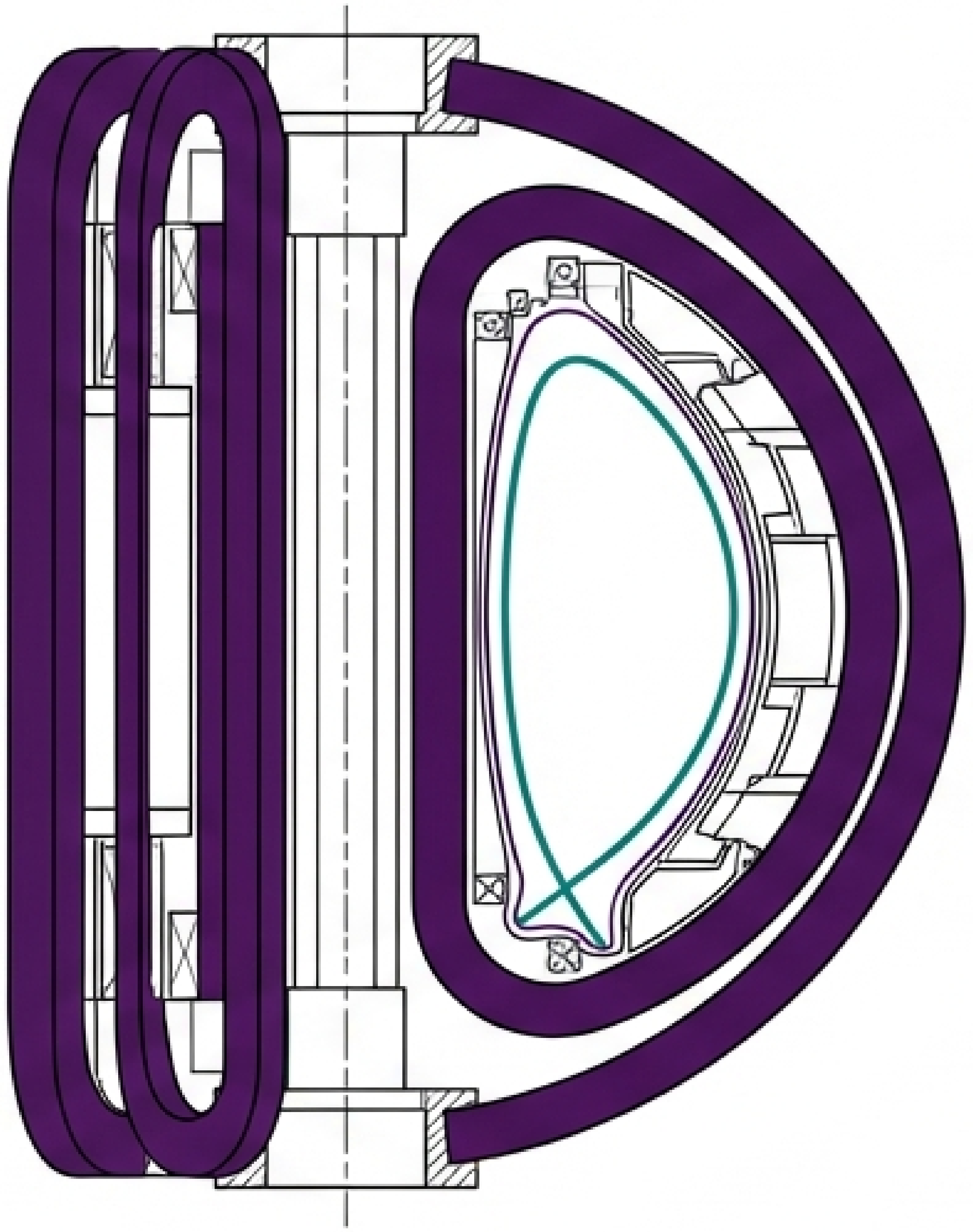
While rotation speed only drops by ~1.6%, the safety margin against mode-locking is dangerously low (0.019), necessitating precise diagnostic capabilities.

ROTATION REDUCTION	SAFETY MARGIN
0.984	**0.019**
Factor (431.8 → 424.9 rad/s)	Relative to unity threshold

MAX CORRECTABLE EF	PARAMETER SCAN
1.69×10^{-4}	NREF (10^{-5}–10^{-3}), β_N (1.0–3.0), Collisionality
$\lvert dB/B_0 \rvert$ limit	

SPARC aims for net energy gain using high magnetic fields and compact geometry

SPARC is designed to achieve net fusion energy gain ($Q > 1$). Success relies on Error Field Correction Coils (EFCC) to manage perturbations. While resonant ($n = 1$) errors are correctable, the "ghost" in the machine—intrinsic NREF—remains a key uncertainty identified by Logan et al. [4].



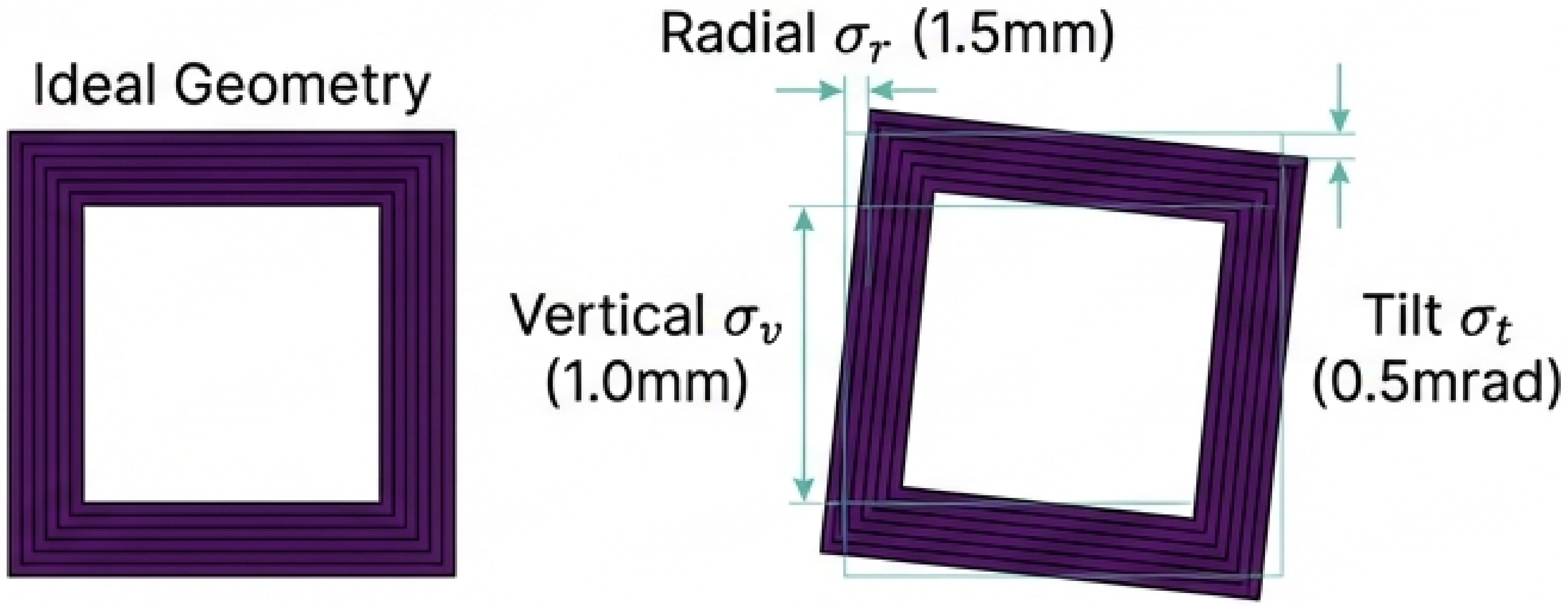
Machine Parameters

Toroidal Field (B_0):	12.2 T
Plasma Current (I_p):	8.7 MA
Major Radius (R_0):	1.85 m
Minor Radius (a):	0.57 m
Elongation (κ):	1.97
Triangularity (δ):	0.54

Intrinsic error fields arise from unavoidable manufacturing tolerances and coil misalignments.

Even with perfect design, physical construction introduces imperfections. We model these "intrinsic" errors based on specific manufacturing tolerances for the 18 toroidal field coils.

These errors add incoherently to the non-resonant harmonics, creating a complex background spectrum of magnetic noise.



Equation (1):

$$\frac{\delta B_{mn}}{B_0} = \sqrt{\left(\sigma_r \frac{a}{R} n C_{mn}\right)^2 + \left(\sigma_t \frac{a}{R} m C_{mn}\right)^2}$$

Equation (2):

$$|\delta B_{mn}^{total}|^2 = |\delta B_{mn}^{coil}|^2 + |\delta B_{mn}^{NREF}|^2.$$

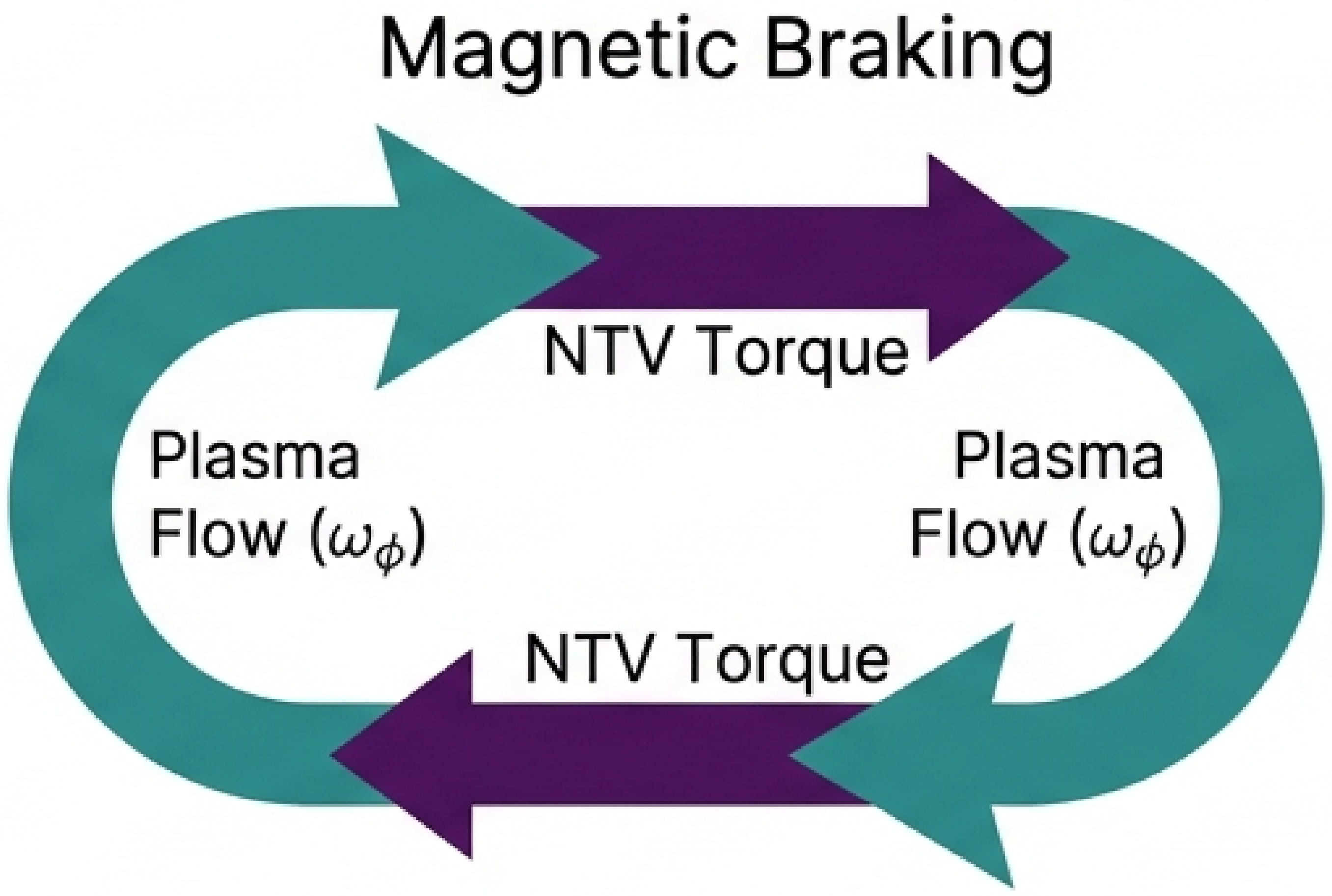
Scientific Editorial

Non-resonant fields drive Neoclassical Toroidal Viscosity (NTV), acting as a magnetic brake.

NREF harmonics do not cause magnetic islands (like resonant fields do), but they degrade confinement via NTV.

NTV torque density depends on the specific transport regime (Superbanana-plateau vs. Ripple-plateau).

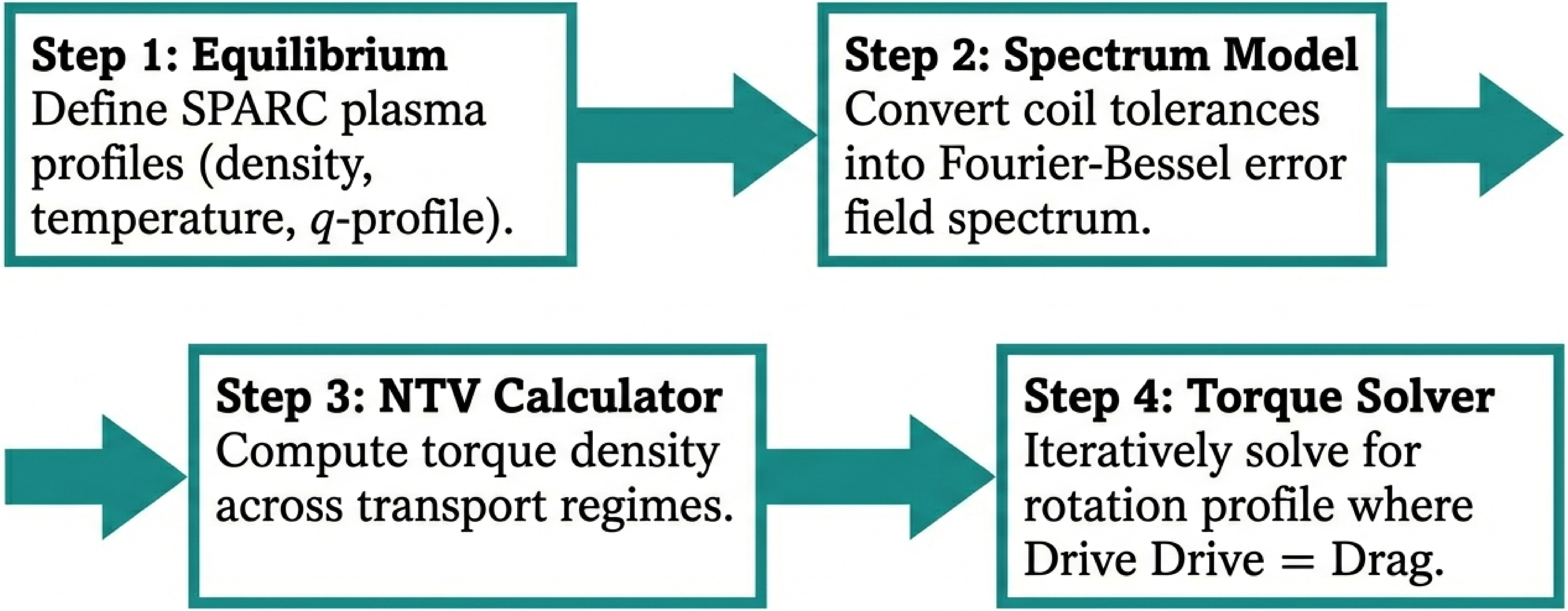
In existing tokamaks (JET, DIII-D), NTV has been observed to lower the error field penetration threshold, making the plasma more susceptible to disruptions.



Equation (3): $T_{NTV} \propto -\nu_{NTV} \cdot p_i \cdot \frac{\omega_\phi}{\omega_t \cdot R_0}$

Effective collision frequency scales with error field spectrum: $\sum |\delta B_{mn}/B_0|^2$

A coupled computational framework integrates spectrum modeling with torque-balance solvers.



Simulation inputs reflect standard high-performance pedestal shapes and safety factors

The model assumes a standard pedestal shape for density and temperature, consistent with H-mode operation. The safety factor (q) profile is critical for determining how magnetic field lines twist.

Core Density (n_{e0}):	$3.1 \times 10^{20}\ \mathrm{m}^{-3}$
Core Electron Temp (T_{e0}):	21.0 keV
Core Ion Temp (T_{i0}):	18.0 keV
Safety Factor (q_{95}):	3.4

$$q(\rho) = q_0 \exp(\alpha \rho^2) \text{ where } q_0 = 1.0$$

Equilibrium rotation is determined by balancing Neutral Beam Injection (NBI) drive against Viscosity.

The code iteratively solves the torque balance equation until a stable rotation profile (ω_ϕ) is found. We assume a fixed NBI power injection as the primary source of rotation.

The Torque Balance Equation (4)

$$T_{NBI} + T_{NTV}(\omega_\phi) + T_{visc}(\omega_\phi) = 0$$

Drive
(25 MW, 200 keV)

Magnetic Drag

Viscous Drag
(χ_ϕ = 0.5 m^2/s)

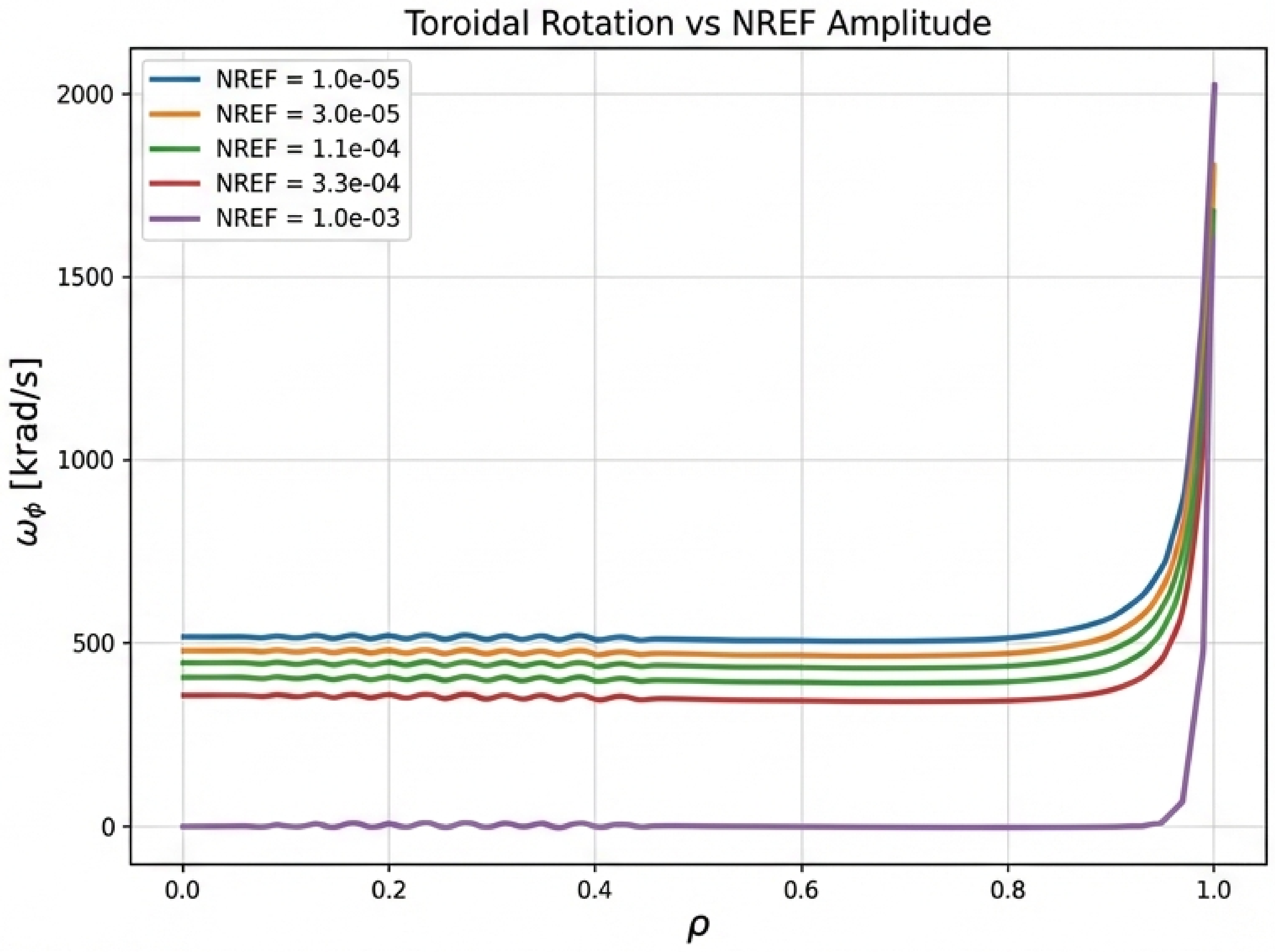
Central rotation decreases monotonically as magnetic ‘**noise**’ amplitude increases.

We scanned NREF amplitude from 10^{-5} to 10^{-3} (relative to B_0). While the rotation profile shape remains consistent, the magnitude drops across the entire radial profile.

Key Results:

Central rotation: $431.80 \rightarrow 424.87$ rad/s.

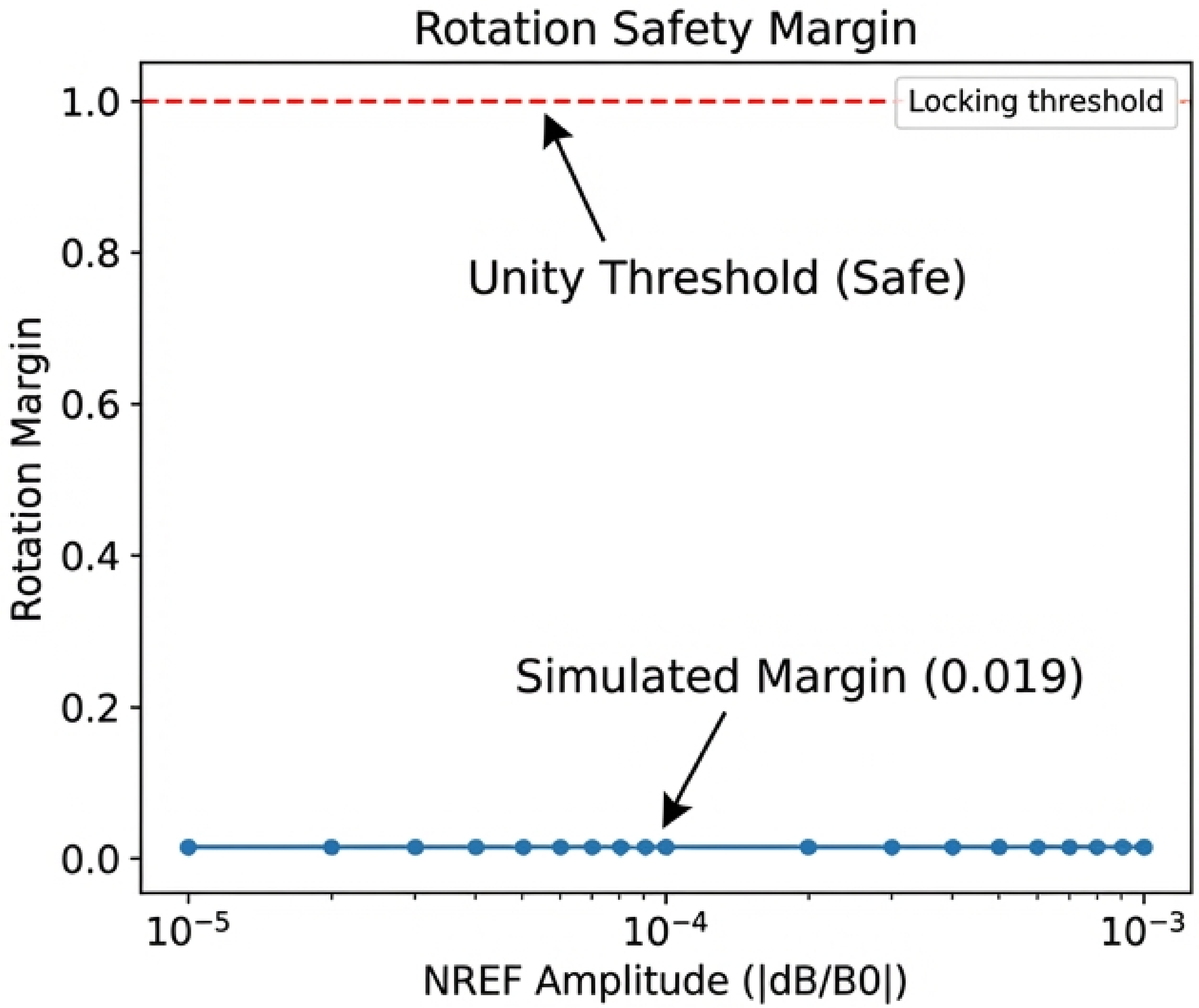
Reduction Factor: 0.984 (1.6% loss)



Despite stable rotation, the safety margin against mode-locking is dangerously low.

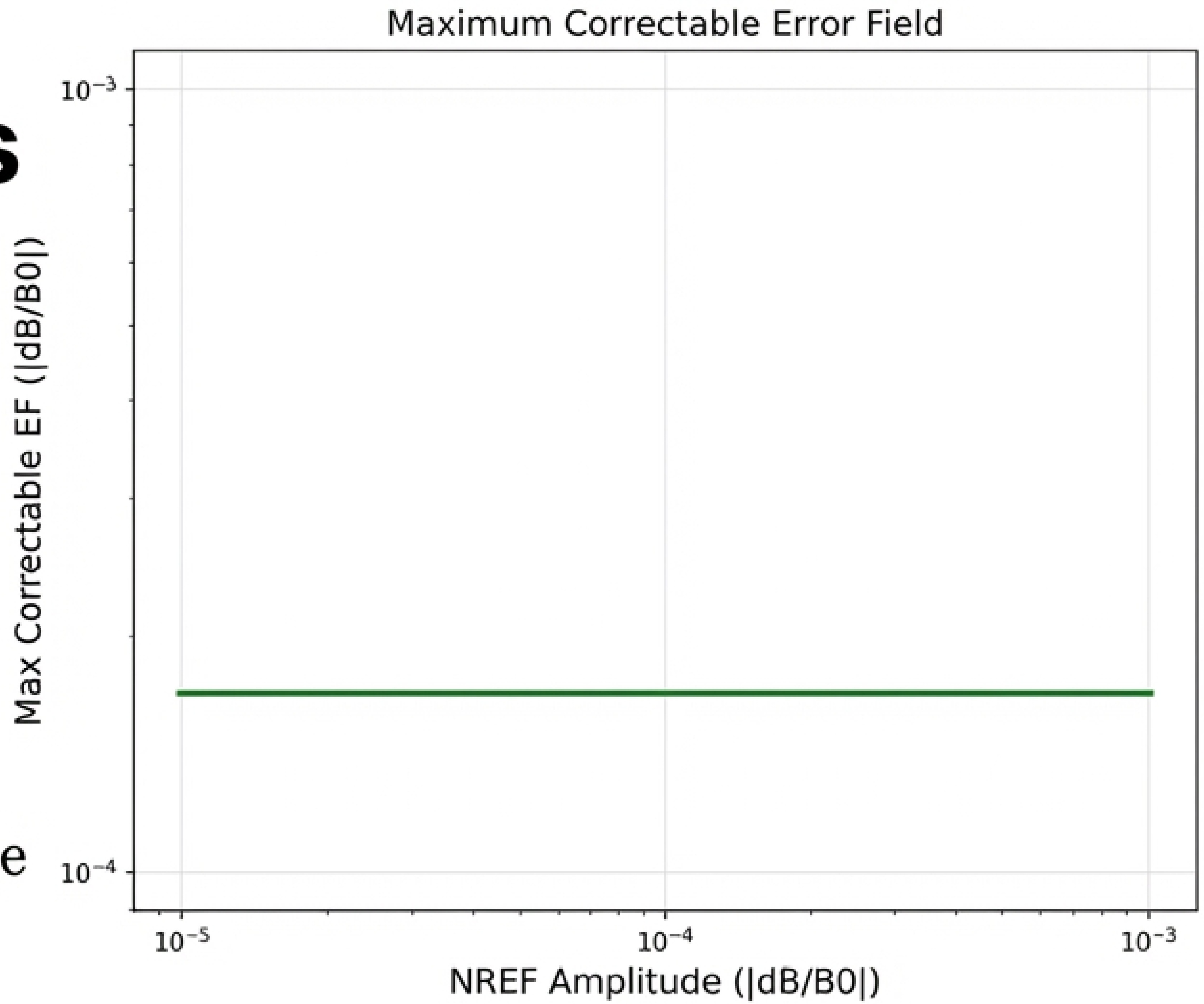
The 'Rotation Margin' defines how close the plasma is to 'locking' (stopping), which typically leads to disruption. A margin of 1.0 is the threshold.

Crucial Finding:
The calculated margin is **0.019**—well below the unity threshold. This implies the system is operating in a regime regime where additional error fields could easily trigger locking.



The maximum correctable error field is capped at 1.69×10^{-4} relative to B_0.

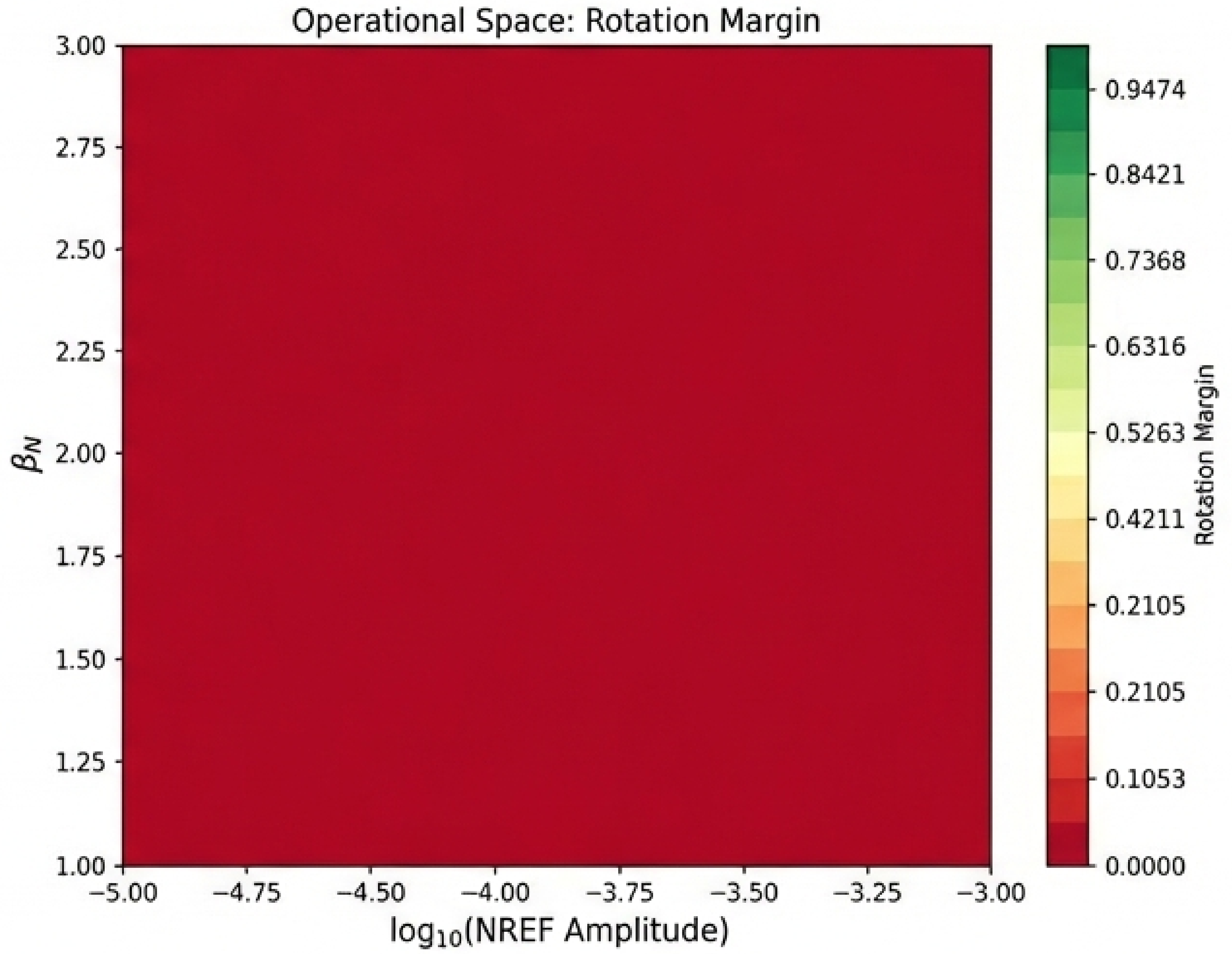
This metric sets the hard requirement for the Error Field Correction Coils (EFCC). If the total error field exceeds this value, the EFCC system may not be able to maintain rotation. The limit is remarkably flat across the tested NREF amplitudes, suggesting it is a fundamental property of the equilibrium.



The low-margin regime persists across the entire target Beta (β_N) operational space.

We mapped rotation margins across Normalized Beta (β_N) from 1.0 to 3.0.

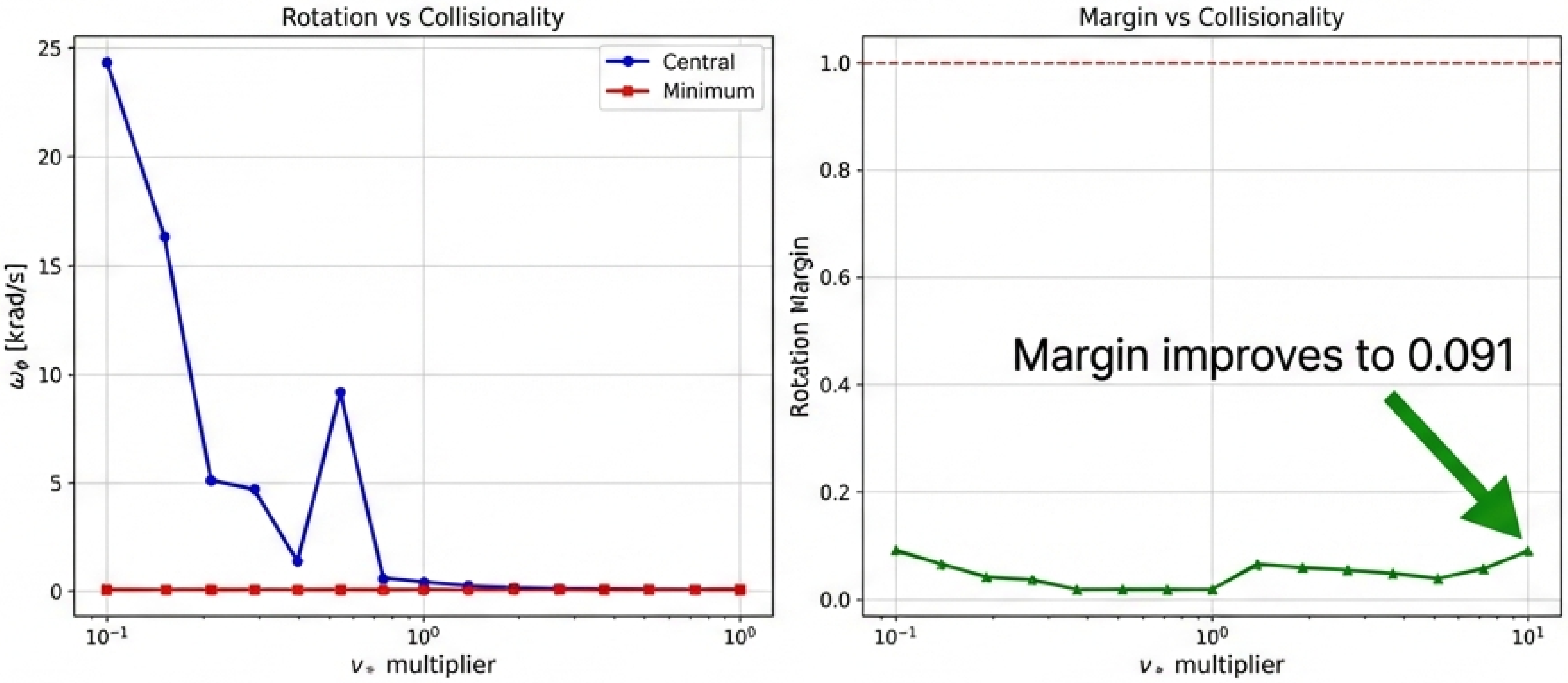
The Result: A 'Red Ocean'
There is no 'safe harbor' where margins naturally improve. The critical NREF amplitude (where magin drops below 1) is extremely low (1.0×10^{-5}) for all tested tested beta values.



Higher collisionality improves safety margins by shifting the NTV torque scaling regime.

Scanning the collisionality multiplier (ν_*) reveals a dependency. As collisions increase (multiplier $0.1 \rightarrow 10$), the margin improves from 0.019 to 0.091.

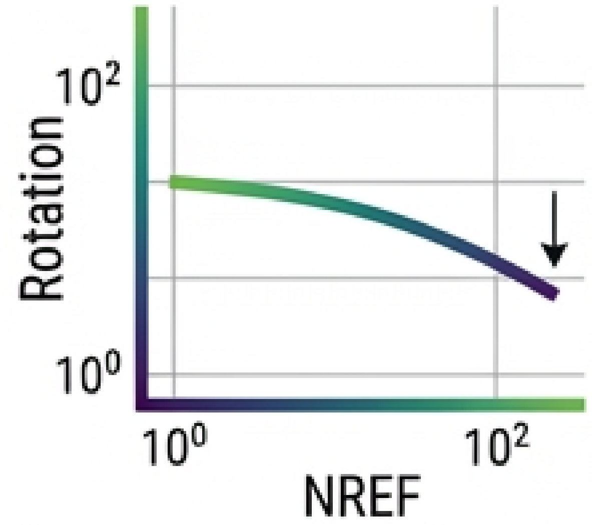
This improvement is due to transitions in the NTV transport regime. However, even at 10x collisionality, the margin remains small (< 0.1).



The system exhibits moderate sensitivity to amplitude, but low absolute stability.

Sensitivity

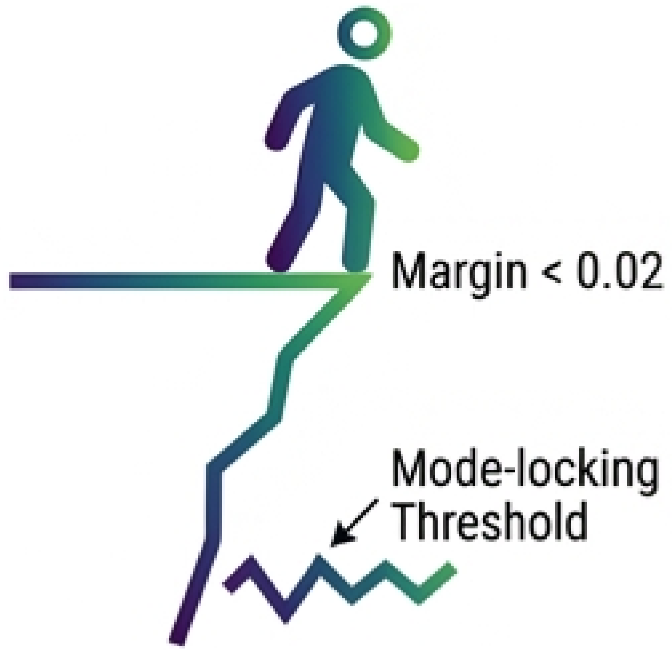
1.6% rotation drop over a 2-order-of-magnitude NREF increase.



Plasma rotation is not catastrophically sensitive to the *magnitude* of the noise.

Stability

Absolute margin of 0.019.



While braking is gradual, the starting point is dangerously close to the mode-locking threshold.

Results define the upper bound for error fields and emphasize transport regime dynamics.

Hard Limits

Finding: Max Correctable EF = $= 1.69 \times 10^{-4}\ (B/B_0)$.

Action: This value serves as a strict 'Do Not Exceed' for the combined intrinsic error + residual corrected error.

Regime Physics

Finding: Margin variability (0.019 to 0.091) with collisionality.

Action: Accurate prediction of SPARC's operational safety requires precise modeling of the specific collisionality regime the plasma will enter.

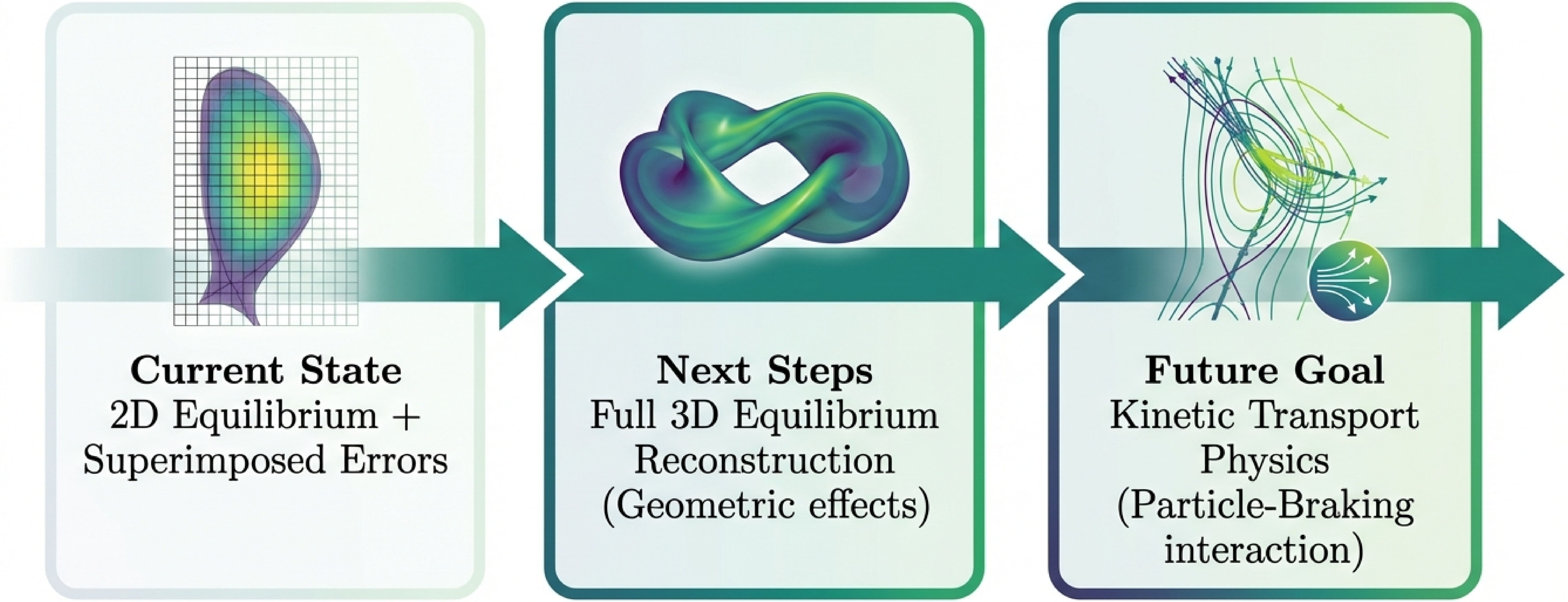
Validating these predictions requires precise diagnostics during SPARC first plasmas.

Assumption Validation
Our results justify the conservative assumption that EFCC-produced fields add to intrinsic NREF.

Diagnostic Need
We need high-fidelity measurements of the NREF spectrum immediately upon startup.

Operational Strategy
EFCC algorithms must be optimized to account for the persistent non-resonant background, not just the $n = 1$ resonant modes.

Future work will incorporate 3D equilibrium reconstruction and kinetic transport models.



This framework provides quantitative bounds for NREF impact, informing critical design margins for SPARC.

Summary of Key Metrics

- **Rotation Impact**: Moderate braking (0.984 factor).
- **Operational Risk**: Low margins (0.019–0.091) require robust correction strategies.
- **Design Constraint**: Max correctable error field established at 1.69×10^{-4}.

References:
[3] Creely et al. (2020) - SPARC Overview
[4] Logan et al. (2026) - EFCC Design
[2] Cole et al. (2007) - NTV Torque